

The Normal Distribution

Recall: 68-95-99.7%

1sd 2sd 3sd

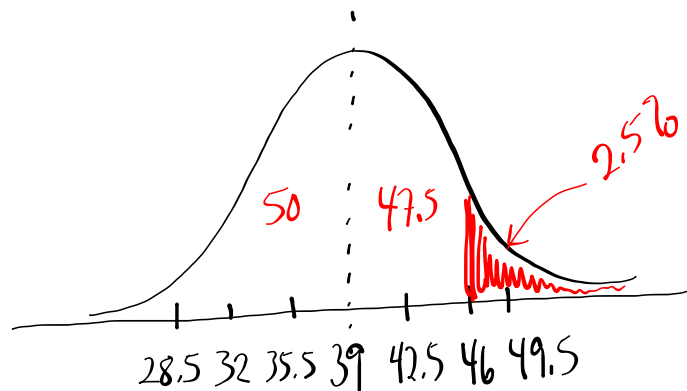
p253/10

normal

$$\mu = 39 \text{ y}$$

$$\sigma = 3.5 \text{ y}$$

How many will
be expected to
live more than 46y?



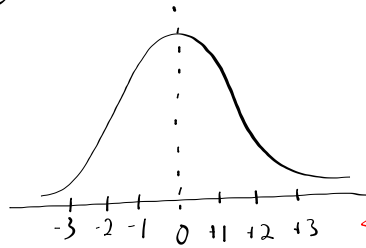
There will be 2.5% of
the dolphins expected to live
longer than 46y.

$$2.5\% \text{ of } 130 = 0.025 (130)$$

≈ 3 dolphins
would be expected
to live longer than
46y

§ 5-5 z-Scores

z-score is the number of standard deviation increments a value is away from the mean.



← these are z-scores

$(-3\sigma, -2\sigma, -\sigma, \mu, +\sigma, +2\sigma, +3\sigma)$

A z-score of +2, means that the value is 2 standard deviations above the mean.

A z-score of -3, means that the value is 3 standard deviations below the mean.

If your value is a "nice" value and at a "nice" increment of the standard deviation away from the mean, you can use 68-95-99.7

What do you do if the value does not fall on a "nice" increment of the standard deviation (i.e. the z-score is not $\pm 1, \pm 2, \pm 3$)?

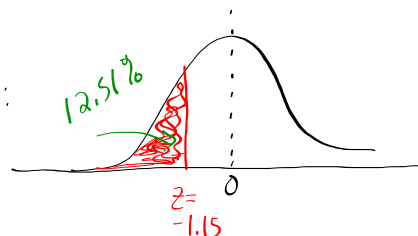
We need to calculate the z-score and use the z-score table

z-score:

$$z = \frac{x - \mu}{\sigma}$$

some value

z-score table:



The z-score table that we are using gives the area under the normal curve to the left of the z-score. (your table -2.99 to +2.99)

Example 1 (p255)

Haley

$$\text{Vancouver} \quad \mu = 25.75 \text{ s} \quad \sigma = 0.62 \text{ s} \quad x = 24.95 \text{ s}$$

$$\text{Lake Louise} \quad \mu = 25.57 \text{ s} \quad \sigma = 0.60 \text{ s} \quad x = 24.77 \text{ s}$$

Which location was Haley's run time better, when compared to the club?

$$\text{Vancouver: } z = \frac{x - \mu}{\sigma}$$

$$z = \frac{(24.95 - 25.75)}{0.62}$$

$$z = \frac{-0.80}{0.62}$$

$$z = -1.29 \leftarrow \text{round to 2 dec places.}$$

↑
This means that Haley's time is 1.29 standard deviations below the mean for the group.

$$\text{Lake Louise: } z = \frac{x - \mu}{\sigma}$$

$$z = \frac{(24.77 - 25.57)}{0.60}$$

$$z = \frac{-0.80}{0.60}$$

$$z = -1.33$$

Haley's time was better than the mean for the group in both locations but it was lowest compared to the group in Lake Louise (lower z-score)

Example 2 (p257)

$\mu = 100$
 $\sigma = 15$
 normal
 $x = 119$

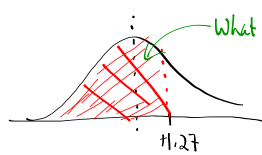
$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{119 - 100}{15}$$

$$z = \frac{19}{15}$$

$z = 1.27$

Look up 1.27
 on z-score table:

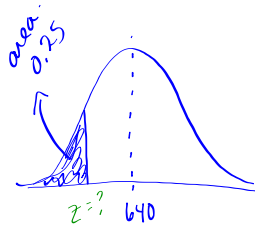


What is the area? 0.8980
 or 89.80%

This means that 89.80% of the population has an IQ less than 119 or 10.20% have an IQ of greater than 119.

Example 3 (p259)

$\mu = 640 \text{ km}$
 $\sigma = 160 \text{ km}$
 area = 25% ^{to the left} or 0.25
 $x = ?$



Look up 0.25 in the body of the z-score table.

This corresponds to a z-score of -0.67

$$z = \frac{x - \mu}{\sigma}$$

$$-0.67 = \frac{x - 640}{160}$$

$$-0.67(160) = x - 640$$

$$-107.2 = x - 640$$

$x = 532 \text{ km}$

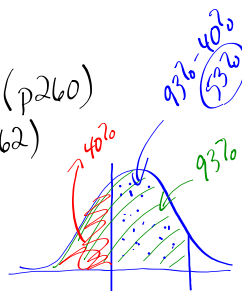
Replace shoes at 532 km.
 (25% of the population will do the same)

TO DO

- ① Look over Example 4 (p260) and Example 5 (p262)

② C4U/p264

③ p264/5-18

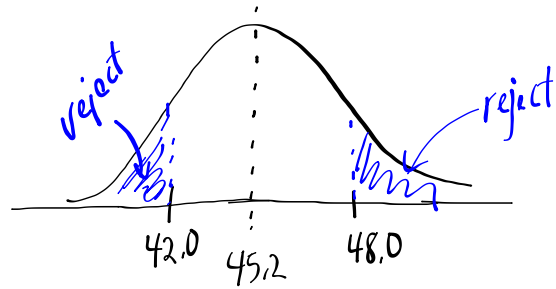


Example 4 (p260)

$$\mu = 45.2 \text{ cm}$$

$$\sigma = 1.3 \text{ cm}$$

normal dist.



shorter than 42.0 cm
longer than 48.0 cm } rejected

How many out of 20 000
will be rejected?

Calculate the z-score
for the two values:

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{42.0 - 45.2}{1.3}$$

$$z = \frac{48.0 - 45.2}{1.3}$$

$$z = -2.46$$

$$z = 2.15$$

find area to
left

find area to
right

$$\text{area} = 0.0069$$

$$\text{area} = 0.9842$$

$$\begin{aligned} \text{area to} &= 1 - 0.9842 \\ \text{right} &= 0.0158 \end{aligned}$$

Total Area

$$= 0.0069 + 0.0158$$

$$= 0.0227 \rightarrow 2.27\% \text{ of the bungee cords} \\ \text{will be rejected.}$$

$$\text{Number rejected} = 0.0227 (20000)$$

$$= 454 \text{ will be rejected.}$$

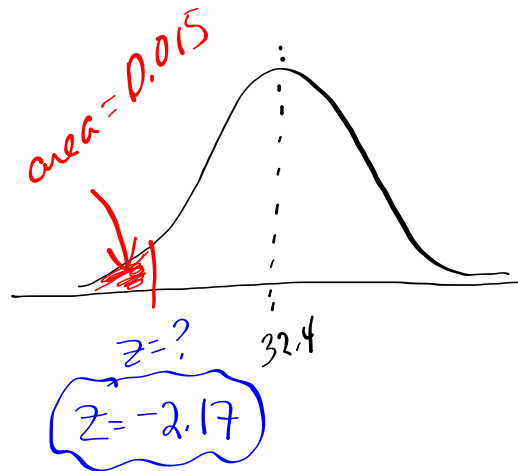
Example 5 (p262)

$$\mu = 32.4 \text{ months}$$

$$\sigma = 6.3 \text{ months}$$

want less than 1.5% repairs

warranty = ?



(What value will give a certain z-score that corresponds to an area of 1.5% or 0.015)

$$z = \frac{x - \mu}{\sigma}$$

$$-2.17 = \frac{x - 32.4}{6.3}$$

$$-13.671 = x - 32.4$$

$$x = ~~18.7~~ \text{ months.}$$

18 The warranty should be for 18 months